⭐ NEWTON'S LAW OF GRAVITATION — COMPLETE STUDY MATERIAL

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🪐 1. Introduction

Gravity is the universe's quiet puppeteer, pulling planets in their orbits and keeping your feet loyal to the ground. Newton transformed this invisible tug into a precise law, stitching together the Earth, Moon, and stars with a single mathematical thread.

🌍 2. Newton’s Law of Universal Gravitation

Statement:
Every particle of matter in the universe attracts every other particle with a force that

1. is **directly proportional to the product of their masses**, and
2. **inversely proportional to the square of the distance between them.**

Mathematically:

$F = G\frac{m_1 m_2}{r^2}$

Where:

- F = gravitational force
- m_1, m_2 = interacting masses
- r = distance between their centers
- G = universal gravitational constant

🧭 3. Universal Gravitational Constant (G)

$$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$$

It is the cosmic glue-strength number. Its small value explains why we don’t feel gravitational attraction toward nearby objects like chairs or books.

📘 4. Vector Form of Gravitational Law

Gravity isn’t just a number; it points.

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^3}\vec{r}$$

The negative sign hints that gravity always **pulls**.

🧩 5. Derivation of Newton's Law of Gravitation

Newton used two grand ideas:

(a) Kepler's Third Law

For a planet of mass m orbiting the Sun of mass M:

$$T^2 \propto r^3$$

$$T^2 = \frac{4\pi^2}{GM} r^3$$

(b) Centripetal Force Requirement

A planet orbiting the Sun:

$$F = \frac{mv^2}{r}$$

Using $v = \frac{2\pi r}{T}$:

$$F = \frac{4\pi^2 m r}{T^2}$$

Substitute from Kepler's equation:

$$T^2 = \frac{4\pi^2}{GM} r^3$$

So,

$$F = \frac{4\pi^2 mr}{\left(\frac{4\pi^2}{GM} r^3\right)} = G\frac{Mm}{r^2}$$

Thus:

$$F = G\frac{m_1 m_2}{r^2}$$

Newton stitched together celestial dynamics and circular motion to reveal the law that rules galaxies.

🪄 6. Gravitational Field (Intensity)

The gravitational field at a point is the force per unit mass placed at that point.

$$\vec{g} = \frac{\vec{F}}{m} = G\frac{M}{r^2}\hat{r}$$

Gravity wants company; it pulls every mass inward.

🏔️ 7. Gravitational Potential

Potential at a distance r from a mass M:

$$V = -\frac{GM}{r}$$

Negative sign means a mass is always bound unless energy is supplied to escape.

🚀 8. Gravitational Potential Energy

For two masses:

$$U = -G\frac{m_1 m_2}{r}$$

Two masses attract and "fall" toward lesser potential.

🌑 9. Motion Under Gravity

(a) Acceleration due to gravity

$$g = G\frac{M}{R^2}$$

Variation with height:

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

Variation with depth:

$$g_d = g\left(1 - \frac{d}{R}\right)$$

🌕 10. Applications of Gravitational Law

- Motion of planets
- Tides
- Satellite orbits
- Escape velocity
- Weight variation on Earth

📜 11. Limitations of Newton's Law

- Fails at high speeds (near light)
- Fails in very strong gravitational fields
- Replaced by Einstein's General Relativity for precision

Yet, Newton's law remains the workhorse of classical mechanics.

✅ 12. Final Notes (Student-Friendly Summary)

Gravity is not a force that pushes—it's a quiet invitation. Everything pulls everything. From galaxies to grains of dust, all matter participates in this cosmic choreography.